

Registration No.:

--	--	--	--	--	--	--	--	--	--

Total Number of Pages: 02

Course: M.Sc.I  
Sub\_Code: FMCC804

8<sup>th</sup> Semester Regular Examination: 2024-25

SUBJECT: MATRIX COMPUTATION

BRANCH(S): M.Sc.I(MC)

Time: 3 Hours

Max Marks: 70

Q.Code: S155

Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)  
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions :

(2 x 10)

- If the matrix  $A$  is ill-conditioned then prove that  $A^T$  is also ill-conditioned.
- State Cholesky decomposition theorem.
- Determine whether the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$  is positive definite.
- Define  $p$ -norm for any real number  $p \geq 1$ .
- Using SVD of  $A$  to deduce the SVD of  $A^T A$ .
- If  $Q$  is orthogonal then show that  $\det(Q) = \pm 1$ .
- Compute the SVD of the matrix  $A = \begin{bmatrix} 3 & 4 \end{bmatrix}$ .
- Write two application of generalized eigenvalue problem.
- How many flops are needed to calculate  $n$  linearly independent eigenvectors of the upper triangular matrix  $T \in \mathbb{C}^{n \times n}$  with distinct eigenvalues?
- If  $\lambda$  is an eigenvalue of a matrix  $A$ , then what can you say about the  $\det(A - \lambda I)$ ?

Part-II

Long Answer Type Questions (Answer Any five)

Q2 a) Prove that if  $A^{-1}$  exists, then there can be nonzero  $y$  for which  $Ay = 0$ .

(5 + 5)

b) Solve the system of equations  $\begin{bmatrix} 5 & 0 & 0 \\ 2 & -4 & 0 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ -2 \\ 10 \end{bmatrix}$  using the column-oriented version of forward substitution.

- Q3** a) If  $A$  is a nonsingular matrix, then show that  $A$  is nonsingular. (5 + 5)
- b) Use inner product formulation to calculate the LU decomposition of the matrix  $A = \begin{bmatrix} 2 & 4 & 2 & 3 \\ -2 & -5 & -3 & -2 \\ 4 & 7 & 6 & 8 \\ 6 & 10 & 1 & 12 \end{bmatrix}$ .
- Q4** a) Give an example of a  $2 \times 2$  matrix, which has only eigenvalue zero but does have two linearly independent eigenvectors. (5 + 5)
- b) Solve the system of equations  $\begin{bmatrix} 0 & 4 & 1 \\ 1 & 1 & 3 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -1 \end{bmatrix}$  by Gaussian elimination with partial pivoting.
- Q5** a) Use QR decomposition to solve the system  $\begin{bmatrix} 2 & 3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 29 \end{bmatrix}$ . (5 + 5)
- b) What does the inverse of a rotator look like? What transformation does it represent?
- Q6** a) Let  $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ . Find a reflector  $\hat{Q}$  and an upper triangular matrix  $\hat{R}$  such that  $A = \hat{Q}\hat{R}$ . (5 + 5)
- b) Let  $v_1 = [3, -3, 3, -3]^T, v_2 = [1, 2, 3, 4]^T, S = \text{span}\{v_1, v_2\} \in \mathbb{R}^4$ . Apply Gram-Schmidt process to  $S$  to obtain an orthonormal basis of  $S$ .
- Q7** a) Find the general solution of the system  $\dot{x}_1 = -9x_1 + 5x_2, \dot{x}_2 = 5x_1 - 8x_2 + 6$ . (5 + 5)
- b) Show that similar matrices have the same eigenvalues.
- Q8** a) Find the singular values and right of the matrix  $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ . Also find the right and left singular vectors. (5 + 5)
- b) Let  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ . Find a space  $S$  that is invariant under  $A$  and is not spanned by eigenvevtors of  $A$ .